Interaction of Signals in Ferromagnetic Microwave Limiters

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Abstract—**A weak secondary signal is partially absorbed in a ferromagnetic microwave limiter that is saturated by a strong primary signal; the absorption is greatest when the two signals are close in frequency. The width of this absorption is determined here, and is found to be proportional to the spin wave linewidth and to the square root of the excess power in the primary signal. The theory of this effect is presented and is found to agree well with experiment.**

I. Introduction

THIS PAPER concerns the absorption of a weak secondary signal in a ferromagnetic microwave limiter that is saturated by a strong primary signal.

A microwave limiter is required to limit the power transmitted through it when a high-intensity signal impinges on it, but to let low-intensity signals pass; at the same time it must, when saturated, transmit weak signals whose frequency is close to that of the saturating signal [1]. Of its nature, however, a limiter must intrinsically be a nonlinear device, and an interaction between neighboring frequencies is to be expected. It is not, therefore, surprising that the secondary signal should be absorbed when its frequency is sufficiently close to that of the main signal, and the bandwidth of this absorption is of considerable technological interest. The following paper gives both calculation and observation of this bandwidth, knowledge of which is important in the design of swept frequency radar systems.

II. Theory

A. Response of the Magnetic System

The motion of the magnetization is here described within the context of Suhl's theory [2] of the subsidiary resonance; quadratic terms in the excursion of the magnetization are included, but no higher order terms.

A microwave magnetic field of frequency ω_0 drives the uniform precession of the magnetization with finite amplitude a. When a becomes sufficiently large, spin waves of frequency $\omega_0/2$ interact with the uniform precession through the quadratic terms in the equations of motion, and $|a|$ reaches a fixed critical value a_c. Any driving magnetic field greater than the critical field h_c required to produce the amplitude a_c will give rise to the amplitude a_c, and a phase lag between field and magnetization will result in absorption of the energy in the field. This energy is converted into spin waves of frequency $\omega_0/2$, and only one particular set of such spin waves achieves a large amplitude.

Manuscript received January 2, 1976; revised July 26, 1976.
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For our purposes we need consider only a single spin wave pair of wave number $\pm \boldsymbol{k}$ and frequency $\omega_k = \omega_0/2$; summations over quadratic terms do not involve spin wave pairs of different wave numbers, so all equivalent spin waves may be lumped together. Let the amplitude of the uniform precession a and of the spin waves be

$$a_0 e^{i\omega_0 t} \qquad b_k e^{i\omega_0 t/2}$$

a_0 and b_k being coefficients that vary slowly with time. From Suhl's theory, these amplitudes have the equations of motion

$$-i\dot{a}_0 + (\omega_0 - \omega_r - i\eta_0)a_0 = -\gamma h + i\rho_k{}^* b_k b_{-k} \quad (1)$$

$$\dot{b}_k + \eta_k b_k = \rho_k a_0 b_{-k}{}^* \quad (2)$$

where h is the amplitude of the applied microwave magnetic field, γ is the gyromagnetic ratio, ω_r is the resonance frequency of the uniform precession, η_0 and η_k are the natural decay constants of the uniform precession and of the spin wave, respectively, and ρ_k is a complicated term whose dimensions are frequency and whose magnitude is of the order of $4\pi\gamma M$, with M being the magnetization of the sample. In particular, one has $\rho_k = \rho_{-k}$.

1) Primary Signal Alone: When only the central signal ω_0 is present, the spin wave system reaches a steady state, $\dot{a}_0 = \dot{b}_k = 0$. From (2), however, it is readily shown that b_k increases exponentially with time if the inequality $|a_0| > \eta_k/|\rho_k|$ is satisfied. Therefore, $|a_0|$ never exceeds the critical value a_c given by the least value of $\eta_k/|\rho_k|$ for all waves with $\omega_k = \omega_0/2$. It is a member of this set of spin waves that must be chosen for $\boldsymbol{k}$.

The amplitude of the steady-state spin waves and of the uniform precession is needed as follows. Let these amplitudes be

$$a_0 = -a_c e^{i\alpha} \qquad b_k = B_k.$$

The critical field needed to produce the amplitude a_c is found from (1) to be given by

$$\gamma h_c = |\omega_0 - \omega_r| a_c. \quad (3)$$

Then, for fields $h > h_c$, one obtains from (1) and (2)

$$a_c = \eta_k/|\rho_k| \qquad \cos\alpha = h_c/h$$

$$|B_k|^2 = \frac{\gamma h}{\rho} \sin\alpha = \frac{\eta_k}{|\rho_k|^2}(\omega_0 - \omega_r)\phi \quad (4)$$

where, in the expression for $|B|^2$, we have dropped a small term of order η_0, and have set

$$\phi = [h^2/h_c{}^2 - 1]^{1/2} = [P/P_{\text{thresh}} - 1]^{1/2}. \quad (5)$$

Here ϕ is an excess power parameter; it involves only the ratio of the incident microwave power P to the threshold power P_{thresh}, which powers are proportional to h^2 and to h_c^2, respectively.

2) *With Secondary Signal:* When a small microwave signal of frequency ω and amplitude h' is superimposed on the main signal, we must set

$$h \rightarrow h + h'e^{i\delta\omega t}, \qquad \delta\omega = \omega - \omega_0. \tag{6}$$

The magnetization now becomes

$$\begin{aligned} a_0 &= -a_c e^{i\alpha} + a^+ e^{i\delta\omega t} + a^- e^{-i\delta\omega t} \\ b_k &= B_k + \beta_k^+ e^{i\delta\omega t} + \beta_k^- e^{-i\delta\omega t} \end{aligned} \tag{7}$$

where a_c, α, and B_k are given by (4); the other terms a^+, β^+, etc., are of first order in h'. To this order, one obtains from (1) and (2)

$$\begin{aligned} (\omega - \omega_r)a^+ &= -\gamma h' + i\rho_k^*(B_k\beta_{-k}^+ + B_{-k}\beta_k^+) \\ (\omega - \omega_r)a^- &= i\rho_k^*(B_k\beta_{-k}^- + B_{-k}\beta_k^-) \\ (i\delta\omega + \eta_k)\beta_k^+ &= \rho_k(a_c e^{i\alpha}\beta_{-k}^{-*} + B_{-k}^* a^+) \\ (-i\delta\omega + \eta_k)\beta_k^- &= \rho_k(a_c e^{i\alpha}\beta_{-k}^{+*} + B_{-k}^* a^-). \end{aligned} \tag{8}$$

The value of a^+ alone gives the susceptibility, since this term alone corresponds to a net magnetization whose frequency is that of the secondary signal. On eliminating other terms and retaining corrections only to first order in a_c^2 and $|B_k|^2$, one obtains for the susceptibility of a material of magnetization M,

$$\chi = Ma^+/h' = \frac{\gamma M}{\omega_r - \omega_0}(\delta\omega - i\eta_k)/[(\delta\omega - 2\eta_k\phi) - i\eta_k].$$

The absorption is given by $\chi'' = \text{Im}(\chi)$, which is

$$\chi'' = \frac{\gamma M}{\omega_r - \omega_0}\frac{2\eta_k^2\phi}{(\delta\omega - 2\eta_k\phi)^2 + \eta_k^2}. \tag{9}$$

This function is multiplied by a factor ϕ, which comes from terms of order $|B|^2$; the theory is valid only to first order in $|B|^2$, so within the limits of the approximations made we are not entitled to retain the frequency shift in the denominator, which is also of order $|B|^2$. This may be verified by solving for χ'' from all of (8), omitting no terms in the algebra; the frequency shift then reverses in sign, but for large values of $\delta\omega$ the result of (9) remains unaltered.

Clearly, the quadratic approximation of Suhl's theory is unable to give the exact shape of the absorption: We are not able to decide if it is or is not asymmetric. However, the width of the absorption in a limiter depends little on the frequency shift (if any), and it is observed that the absorption is more or less symmetrical in single crystal limiters. Rather than carry out a heroic calculation to sixth order in the spin wave amplitudes, we shall simply take the limiting form of the susceptibility for large values of $\delta\omega$, which is

$$\chi'' = \frac{\gamma M}{\omega_r - \omega_0}\cdot\frac{2\eta_k^2\phi}{(\delta\omega)^2}. \tag{10}$$

B. Absorption of Energy

Successive elements of a microwave limiter absorb separate increments of the incident power. At high powers, all elements may be subject to a microwave magnetic field above the critical field, while at lower powers only the first one or two elements may be saturated. We must therefore take account of the variation in driving power along the length of the limiter; it will be assumed that the variation is continuous.

Let the primary microwave field at some point x on the limiter be $h(x)$. The mean rate $\overline{W}$ at which work is done on a volume V of magnetic material is

$$\begin{aligned} \overline{W} &= V\langle \boldsymbol{M}\cdot\boldsymbol{h}\rangle_{\text{av}} = \omega_0 a_c MhV\sin\alpha \\ &= \frac{\omega_0\gamma Mh_c^2V}{\omega_r - \omega_0}\phi \end{aligned} \tag{11}$$

where ϕ is the excess power parameter defined in (5).

But the traveling power P is proportional to h^2, and

$$\frac{dP}{dx} = -C\overline{W} \tag{12}$$

C being some constant. Therefore, from (5) and (11),

$$\frac{d\phi}{dx} = -\frac{1}{2}\frac{\omega_0\gamma MV}{\omega_r - \omega_0}C \tag{13}$$

i.e., the excess power parameter ϕ drops linearly along the limiter until it falls to zero; thereafter it remains zero.

The weak secondary signal is absorbed only in the region where $\phi > 0$. At some point x on the limiter, let the secondary signal have amplitude h'; the rate at which work is done on a volume V of magnetic material is

$$\begin{aligned} \overline{w} &= V\langle \dot{\boldsymbol{m}}^+\cdot\boldsymbol{h}'\rangle_{\text{av}} \\ &= \omega Vh'^2\chi''. \end{aligned}$$

But h'^2 is proportional to the traveling power p in the secondary signal. From (12) one has

$$\frac{dp}{dx} = -C\overline{w} = -\omega VC\chi''p.$$

We are interested only in the region where $\phi > 0$; on changing from x to ϕ as a variable, one obtains from (13)

$$\frac{d\ln p}{d\phi} = -2\frac{\omega_r - \omega_0}{\gamma M}\chi''.$$

The attenuation of the secondary signal is therefore given by

$$\begin{aligned} \ln\left(\frac{p_{\text{in}}}{p_{\text{out}}}\right) &= 2\frac{\omega_r - \omega_0}{\gamma M}\int_0^\phi \chi''(\phi')\,d\phi' \\ &= \frac{2\eta_k^2\phi^2}{(\delta\omega)^2} \end{aligned} \tag{14}$$

where χ'' has been taken from (10).

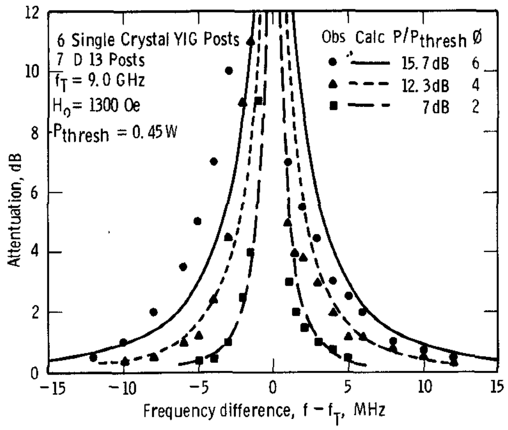

Fig. 1. Dependence of the additional attenuation of the weak signal on frequency, for three power levels of a saturating signal of frequency f_T = 9 GHz.

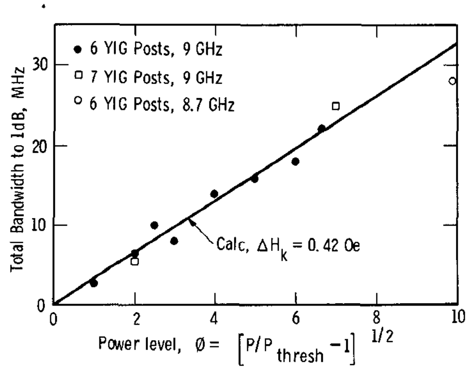

Fig. 2. Total bandwidth of the absorption (to the 1-dB level) as a function of input power.

III. Experimental Results

Frequency selectivity has been measured in microwave power limiters whose basic geometry was similar to that described by Carter and McGowan [3]. Each limiter was an alternating array of single crystal YIG posts and of dielectric posts with $\varepsilon_r = 13$ placed against the side wall of a waveguide of internal dimensions 1 × 2.25 cm. The ferrite was biased into the subsidiary resonance mode by a steady magnetic field of approximately 1300 Oe, this field being parallel with the microwave electric field in the waveguide.

Two such limiters were used. One, used only in two preliminary experiments, had seven YIG posts and eight dielectric posts. The other, used in most of the experiments, had six YIG posts and seven dielectric posts. For this limiter the limiting threshold (defined as the power at which the attenuation was 1 dB above that of a weak signal) was 0.45 W at 9 GHz and 0.3 W at 8.7 GHz. There were also two sources of high microwave power used: In most of the experiments (up to 20 W) the source was an ultrastable microwave oscillator driving a traveling wave tube amplifier at 9 GHz; in a single experiment at higher power the source was a magnetron operated at 8.7 GHz. The magnetron was considerably noisier than the first source.

The sequence of the experiment was as follows. A below-threshold signal of frequency f and 1-mW power was applied to the input port of the limiter, and the output was observed on a spectrum analyzer. An above-threshold signal at $f_T = 9$ GHz (or $f_T = 8.7$ GHz for the magnetron source) was then applied; the weak signal now suffers an increased attenuation. This additional attenuation was recorded for various power levels at 9 GHz, and at various frequencies f of the weak signal.

Equation (14) is, in principle, accurate only for large values of $\delta\omega$; the additional attenuation is then

$$p_{\text{out}}/p_{\text{in}} = (\delta\omega)^2/[(\delta\omega)^2 + 2\eta_k^2\phi^2]. \quad (15)$$

Experimental and theoretical results at three power levels are compared in Fig. 1; in the calculation it was taken that $\eta_k = 3.66 \times 10^6\ \text{s}^{-1}$ (0.583 MHz), a value obtained from a best fit to the 1-dB points at all power levels (see the following). Agreement between theory and experiment is fair at larger frequency differences when the attenuation is low, as is to be expected from the approximations in the theory. Near the center, however, the absorption is markedly stronger for negative values of $f - f_T$ than for positive values. The possibility of such an asymmetry in the absorption was suggested after (7), but the extent of the asymmetry cannot be calculated in a second-order theory.

The calculation should give the total width of the absorption, though not its exact shape. The total width at the 1-dB level of attenuation is

$$\Delta f_{1\text{ dB}} = 0.444\gamma\Delta H_k\phi \quad (16)$$

where ΔH_k is the spin wave linewidth as conventionally defined,

$$\Delta H_k = 2\eta_k/\gamma.$$

The difference between the two frequencies, one above and one below f_T, at which the weak signal attenuation is 1 dB was measured at various values of the power parameter ϕ, for both of the limiters mentioned previously. A plot of bandwidth versus ϕ is shown in Fig. 2. The straight line is a least squares fit to the 9-GHz (clean source) data, and corresponds to a spin wave linewidth $\Delta H_k = 0.41$ Oe. This linewidth agrees well with other published results for YIG; Fletcher *et al.* [4] find values of ΔH_k between 0.41 and 0.64 Oe, while Schlömann *et al.* [5] find ΔH_k between 0.35 and 0.52 Oe.

IV. Conclusion

The preceding account gives a simple picture of the interaction of a weak signal with a saturating signal in a ferromagnetic microwave limiter. The large uniform precession of the magnetic moment due to the strong signal is modulated by the weak signal, so the rate at which high-amplitude spin waves are generated by the uniform precession is also modulated. The resultant modulation in the spin wave amplitude reacts on the uniform precession and causes a power loss at the frequency of the weak signal. The calculated bandwidth of this loss depends only on the spin wave linewidth and on the power in the saturating signal; the theory agrees well with observation.

This work has obvious applications to radar systems, especially where the frequency is swept; it also provides a novel and sensitive way to determine spin wave linewidths.

ACKNOWLEDGMENT

The authors wish to thank Dr. R. W. Patterson for his advice during this enquiry.

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Computer-Aided Synthesis of the Optimum Refractive-Index Profile for a Multimode Fiber

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***Abstract*—In a multimode optical fiber, the so-called multimode dispersion (mode-delay difference) is the principal cause that widens the transmitted pulse. The multimode dispersion can be controlled by the refractive-index profile. However, the optimum profile that minimizes the multimode dispersion has not yet been determined.**

This paper describes the computer-aided trial-and-error synthesis of the optimum refractive-index profile. It is shown that the group delay is reduced to about 10^{-3} times the value obtained with the uniform core fiber, to about 10 ps/km. This value is comparable to the material dispersion obtained with an ordinary fused-silica fiber and a typical semiconductor laser. It is also shown that the optimum profile is a smoothed W-shaped one.

I. INTRODUCTION

SEVERAL types of permittivity profiles have been proposed as the optimum profile that minimizes the multimode dispersion (mode-delay difference) of an optical fiber [1]–[4]. In those proposals, however, the permittivity in the core is assumed to be proportional to r^α, where r is the radial coordinate and α is an arbitrary positive quantity. Therefore, the obtained profile cannot be the genuine optimum.

This paper describes an approach to the genuine optimum. We express the permittivity in the core by a power series in terms of r, and use the variational method [5] to obtain the delay time of each propagation mode. Next we compute the variance of the delay time, i.e., the group delay. Then we modify the permittivity profile so as to decrease the group delay toward its minimum. We repeat the aforementioned process of analysis, estimation, and modification until we obtain the optimum permittivity profile with which the group delay is minimized. The whole process of such trial-and-error synthesis is performed in the computer.

The example of the synthesis described in Section V of this paper is the synthesis of the optimum profile for a fiber in which ten LP modes propagate. The same method can, of course, be used for any number of modes. It is shown that the group delay can be reduced to about 10^{-3} times the value obtained with the uniform core fiber, to about 10 ps/km, and that the optimum profile is a smoothed W-shaped one. This result substantiates the validity of the proposals made by Suematsu and Furuya for slab waveguides [6] and the present authors [3].

Manuscript received March 26, 1976; revised July 19, 1976.
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II. RESTRICTING CONDITIONS

We assume that the refractive-index distribution is axially symmetric, and that the quantities listed as follows remain constant in the course of the optimization:

1) wavelength of light λ;
2) the maximum refractive index n_1 in the core and the refractive index in the cladding n_2;
3) number of propagating LP modes M.

Note that the core radius a is not fixed. The relative difference of the refractive indices, which is defined conventionally as

$$\Delta = \frac{(n_1{}^2 - n_2{}^2)}{2n_1{}^2} \doteqdot \frac{(n_1 - n_2)}{n_1} \tag{1}$$

also remains constant from the preceding condition (2).